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Nonlinear mechanics of fluidization of beds of spherical particles

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Experiments on fluidization with water of spherical particles falling against gravity in columns of rectangular cross-section are described. All of them are dominated by inertial effects associated with wakes. Two local mechanisms are involved: drafting and kissing and tumbling into stable cross-stream arrays. Drafting, kissing and tumbling are rearrangement mechanisms in which one sphere is captured in the wake of the other. The kissing spheres are aligned with the stream. The streamwise alignment is massively unstable and the kissing spheres tumble into more stable cross-stream pairs of doublets which can aggregate into larger relatively stable horizontal arrays. Cross-stream arrays in beds of spheres constrained to move in two dimensions are remarkable. These arrays may even coalesce into aggregations of close-packed spheres separated by regions of clear water. A somewhat weaker form of cooperative motion of cross-stream arrays of rising spheres is found in beds of square cross-section where the spheres may move freely in three dimensions. Horizontal arrays rise where drafting spheres fall because of greater drag. Aggregation of spheres seems to be associated with relatively stable cooperative motions of horizontal arrays of spheres rising in their own wakes.

1. Introduction

In this paper we describe some experiments on the fluidization of spherical particles between two parallel glass plates and in a Perspex column with a rectangular cross-section. Some experiments between parallel plates were performed in beds of different width with spheres whose diameters were slightly smaller than the gap between the plates. We say that this fluidization of a single layer of particles is 'two-dimensional'. To our knowledge, the experiments here and in the papers of Volpicelli, Massimilla & Zenz (1966) and Garside and Al-Dibouni (1973) are the only ones using two-dimensional beds of a single layer of particles. Some of the observations which we shall make about the dynamics of fluidization in these beds can also be inferred by careful examination of photographs presented by Volpicelli *et al.*, though they did not make these inferences. They showed that fluidization in two-dimensional beds satisfies the fluidized-bed correlations of Richardson & Zaki (1954) with a shifting of curves of correlation due to wall effects.

The fluid dynamics of two-dimensional fluidization of spheres is of intrinsic interest and is also useful as a diagnostic tool for three-dimensional fluidization. It is certain that there are some wall effects in two-dimensional beds. However, the dominant features of fluidization so clearly evident in the two-dimensional beds were also observed in three-dimensional beds at all Reynolds numbers for which fluidization was possible. The three-dimensional beds are the parallel plates with smaller spheres (two-layer beds) and columns with rectangular and square cross-sections, using spheres of different densities and sizes.

Different flow regimes were observed in the experiments. All of them are dominated by inertial effects associated with wakes. A basic mechanism of fluidization involves pairwise coupling of spheres whose line of centres is parallel to the stream. We describe the dynamic scenario associated with this pairwise coupling as drafting, kissing and tumbling. The second sphere is drafted into the wake of the first sphere: they kiss, then tumble. The falling motion of kissing spheres whose centres are parallel to the stream is unstable to couples of the type which turn streamlined bodies broadside on. Neighbouring spheres with centres aligned cross-stream which appear to have the greatest stability are also a characteristic feature of the fluid dynamics of fluidized beds. The robust stability of the cross-stream alignments are particularly dramatic to a 'two-dimensional' bed of a single layer of particles. In this way drafting and kissing can lead to aggregation. The aggregates are closely packed arrays of spheres which can align in contact along vertical lines through their centres. Such aggregates are a characteristic feature of a two-dimensional fluidized bed, and they can take form as propagating or even as standing 'shock waves' of aggregates separated by clear water. Propagation of stable cross-stream arrays also occurs in the three-dimensional case. We see cases where the propagation occurs as a result of drafting, kissing and tumbling spheres collecting at the top and leaving the bottom of the aggregates.

The clear-water regions between aggregates will be called voidage cracks. Voidage cracks, called 'bubbles', are a characteristic feature of fluidization of beds of particles with air. Bubbles look much different than voidage cracks and the dynamics of fluidization with air must be hugely different than fluidization with water. The change of momentum in the liquid of liquid–solid systems is as important as the changes in the momentum of the particles, because their densities are not greatly different. The literature on fluidization is abundant with empirical and theoretical analysis of crack formation in fluidized beds. The latest works on the subject, such as those by El-Kaissy & Homsy (1976), Homsy, El-Kaissy & Didwania (1980) and Liu (1982, 1983) hold that the crack formations can be traced to voidage instability waves. Though we are not yet committed to any theoretical model of a fluidized bed, our observations indicate that aggregation associated with drafting, kissing and tumbling can give rise to the propagation of voidage cracks. This type of propagation can be viewed as the shock waves of the effective density of spheres which were described in the works by Wallis (1962), Verloop & Heertjes (1970) and Fanucci, Ness & Yen (1979). In the fluidization of a single layer of particles, suction in the wakes of spheres in relative motion leads to aggregation due to repeated capture of spheres and even to standing shock waves of closely packed spheres which percolate along vertical lines, as in figure 1.

The topology of sphere aggregation in fluidization is a functional of the dynamics. Rowe (1961) and Rowe & Henwood (1961) studied drag forces on spheres in fixed aggregates. Unfortunately, they pushed their flow through the unnatural array of spheres shown in figure 1 when U is turned through 90°. The hexagonal arrangements of similar spheres used by Richardson & Meikle (1964) are also misleading, because they artificially constrain the spheres.

Many investigators have reported the formation of natural clustering of particles in both gas and liquid–solid systems. As early as in the work of Wilhelm & Kwauk (1948) and in the sedimentation experiments of Kaye & Boardman (1962), the formation of particle clusters has been reported. Experimental evidence of the importance of aggregates formed from drafting, kissing and tumbling was given by

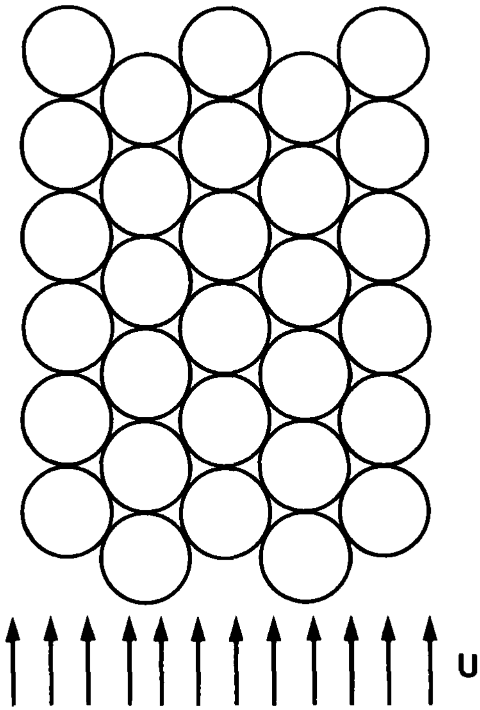


FIGURE 1. Closely packed spheres contacting on vertical lines of cluster parallel to the stress.

Happel & Pfeffer (1960). Their study was restricted to free-falling spheres along the axis of a cylindrical glass column of liquid with Reynolds numbers in the range of 0.3 to 0.7. Even at these low Reynolds numbers wake effects are important, even dominant. Happel & Pfeffer conclude their experimental study with the observation that 'it is possible the spheres suspended in random orientation may not maintain their positions relative to each other. Perhaps the formation of doublets (kissing spheres), and their corresponding higher velocities of fall, has been one of causes for the wide discrepancies in the presently available fluidization data.'

Recent experimental work of Tsuji, Morikawa & Terashima (1982) on the hydrodynamic interaction between two spheres is relevant to our investigation. They used the pendulum method to measure the drag on the sphere and flow visualization. The pendulum method restrains the sphere to move like a pendulum at the end of a string. The method was extended to study flow interaction: a dummy sphere is set up in the front or rear of the test sphere, or a group of spheres connected by a rod in the transverse direction is hung by two strings. They used both streamwise and crosswise arrangements and determined the corresponding effects on the drag and vortex patterns. Their results agree with our findings on the existence of the suction in the wakes of spheres in relative motion even at low Reynolds numbers, when a single sphere does not shed vortices, and with our observation of increased drag on the cross-stream arrays.

The compelling experimental demonstrations of wake-dominated nonlinear mechanisms for fluiding beds of particles ought to be addressed in the hydrodynamic modelling of fluidizing and sedimentation.

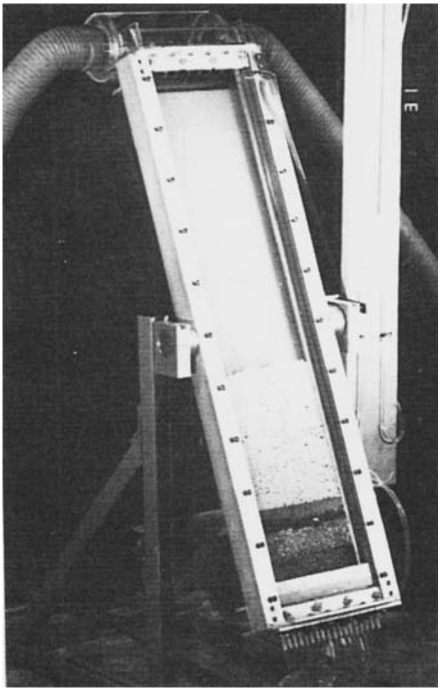


FIGURE 2. 'Two-dimensional' apparatus.

2. Experimental parameters

The two-dimensional bed exhibited in figure 2 was made of two parallel glass plates 0.285 in. apart with a 75 cm test section. The width was adjustable. The glass plates were framed in U-shaped beams, and changes of the bed thickness within ± 0.007 in. were made possible with the use of adjustment screws so as to ensure an even distribution of the water flow throughout the bed. The working fluid was water supplied from a pressurized water tank through a needle valve. This valve gives fine control of the flow rate so that a fairly steady working pressure was available at the entrance of the bed. The maximum total throughput of the 5 hp centrifugal-pump water-supply system was approximately 50 l/min. An equalizing section consisting of a 6 in. layer of packed glass beads followed by a 2 in. layer of metal foam was placed upstream of the test section. Pressure taps attached to the rear plate of the bed were located right above the equalizing section and at the top of the test section. The water flowing from the top of the bed was collected in a trough open to the atmosphere. The flow rates were measured with a Micro Motion Mass Flow Meter, and the pressure drop across the entire bed was measured with piezometer tubes. The two-dimensional bed could be tilted at any angle from the vertical. This allows one to adjust the effective force of gravity.

The three-dimensional bed is exhibited in figure 3. The rectangular cross-sections used in this Perspex-plate apparatus were 3 in. × 2 in. and 3 in. × 3 in. The square cross-section, used for plastic beads, did not allow through flow of a magnitude sufficient to fluidize the heavier glass beads. The smaller cross-section was used in the experiments on fluidization of glass beads. This bed had an equalizing section

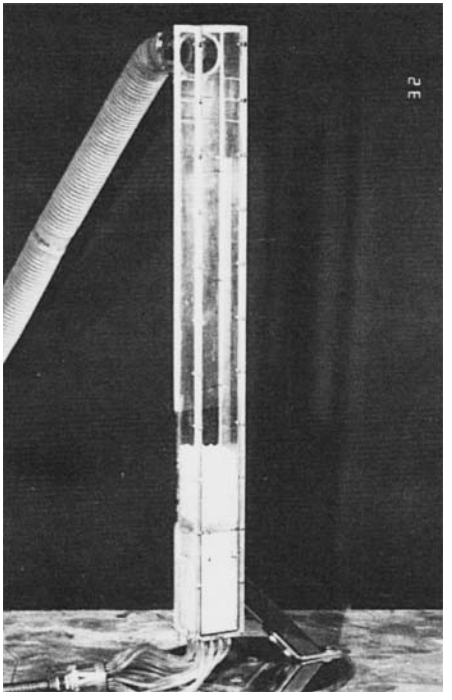


FIGURE 3. 'Three-dimensional' apparatus.

Particles	Diameter D (in.)	Density ρ (g/cm^3)
Glass	0.231	2.462
	0.117	2.462
Plastic	0.251	1.119

TABLE 1

composed of a 4 in. layer of honey-comb aluminium followed by a 1 in, layer of metal foam.

Both beds could be illuminated from behind as well as from the front with 4 high-intensity 650 W quartz flood lamps. Illumination of the two-dimensional bed was made through an opaque Perspex plate graduated with a metric scale on its left-hand side for measuring the bed height. The three-dimensional beds were illuminated from the front.

Flow visualizations were recorded using still 35 mm photography and conventional video-cassette recording. Motion analysis was made possible with the help of a Spin Physics Motion Analysis System which incorporates a high-speed video camera that can take up to 2000 frames per second. In order to enhance the contrast against a black background, the glass beads were etched in a solution of hydrofluoric acid. Some plastic beads were dyed in a solution of red oil dye and acetone in order to be better able to follow the motion of one single particle in the bed. The bed width was adjustable. Average properties of the spheres are given in table 1.

In the two-dimensional bed, bed widths of 3, 6, and 8 in. were used. The flow rates

Particles	Diameter (in.)	Bed	Velocity (m/s)
Glass	0.231	two-dimensional	0.341
	0.117	two-dimensional	0.337
		three-dimensional	0.551
		three-dimensional	0.372
Plastic	0.250	two-dimensional	0.094
		three-dimensional	0.154

TABLE 2

used varied from 5.2–21.5 l/min for fluidization with glass beads and from 1 to 3.1 l/min for the plastic beads. In the three-dimensional bed, the maximum flow rate used was about 48 l/min for dimensional of glass spheres. Smaller flow rates were sufficient for the dimensional of plastic spheres.

The voidage ϕ is the fraction of water in the total volume occupied by the fluidized bed. The total volume is the box which is defined by the portion of the bed between the two planes perpendicular to the flow direction and tangent to the highest and lowest sphere in the bed. When using the smaller glass particles in the two-dimensional bed and for all the experiments in the three-dimensional bed, the void fractions were calculated from the volume of the particles determined by a careful measurement of the water added up to the least possible static bed height and from the known total volumes, the total volume changes as the bed expands.

The measured vertical free-fall velocities in water of the glass and plastic beads, given in table 2, were measured within a precision better than 1 %.

The Reynolds number is $Re = DU_s/\nu$ where D is the diameter of the sphere, ν is the kinematic viscosity and U_s is the superficial velocity of the fluid based on the net volume flux of water across a plane perpendicular to the flow direction. The Reynolds numbers of our experiments were always greater than 270, making our flow regimes definitely non-Stokesian. Potential flow analysis of the dynamics of fluidization is equally deficient, because the dynamics is controlled by suctions in the wakes of spheres in relative motion. For example, analysis shows that two spheres in steady potential flow will attract one another when they move perpendicular to their line of centres and they will repel one another when they move parallel to their line of centres (Lamb 1938, p. 191). In fact, these predictions do not apply to flows with wakes. It seems probable that many features of the flows we observed could be explained by a proper introduction of vortical regions (wakes) into other-wise potential flow. In our experiments, spheres moving broadside-on do not attract and spheres following along their line of centres do, a result already known even for weak inertial effects, as the Oseen's equations show (Oseen 1927, pp. 199 ff.).

3. Analysis of drafting, kissing and tumbling

Fundamental features of the fluid dynamics may be observed by fluidizing two spheres. Figure 4 (*a*, *b*) shows two plastic spheres suspended in a rising stream in the two-dimensional bed. The bed was inclined 23° 35′ from the vertical. This inclination introduced a gravity force component acting on the spheres normal to the walls of the channel, which has increased the stabilizing effects of the wall friction. We will show later that this wall friction does not significantly change the particle

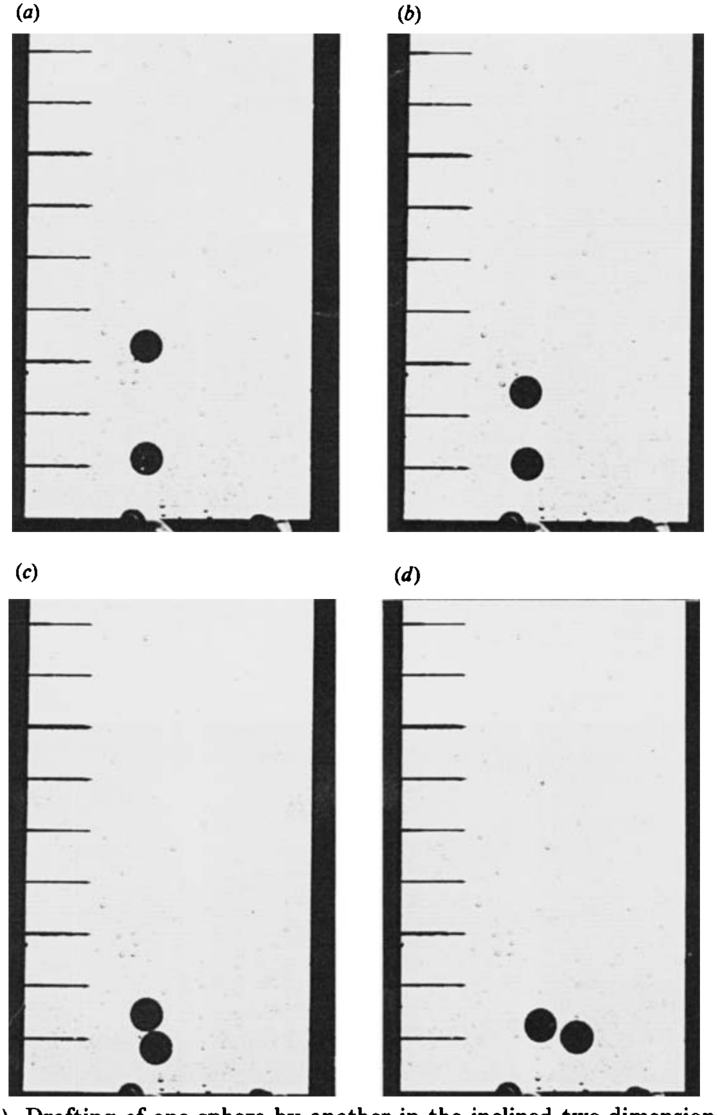


FIGURE 4(*a*, *b*). Drafting of one sphere by another in the inclined two-dimensional bed, the line of centres lies along the stream. Both spheres are falling, but the second is falling faster than the first, accelerating toward it. The bed is inclined 23° 35′ from the vertical. $Re = 700$. (*c*) Kissing after drafting. (*d*) Tumbling after kissing.

interactions. The Reynolds number was 730. In figure 5(*a*) the same plastic spheres were suspended in a vertical channel. The Reynolds number was slightly smaller than in the inclined channel. A single sphere is in a force equilibrium between the net weight of the sphere and the drag on it. If the two spheres do not interact, they are nearly in equilibrium and drift slowly from place to place. One sphere always drifts into the wake of the other. The second sphere which is now on the line of centres parallel to the stream begins to acelerate, first slowly, then rapidly, to the falling first sphere. This can occur when the centres of spheres are even five or six diameters apart. This is the drafting part of the drafting, kissing and tumbling scenario as shown in

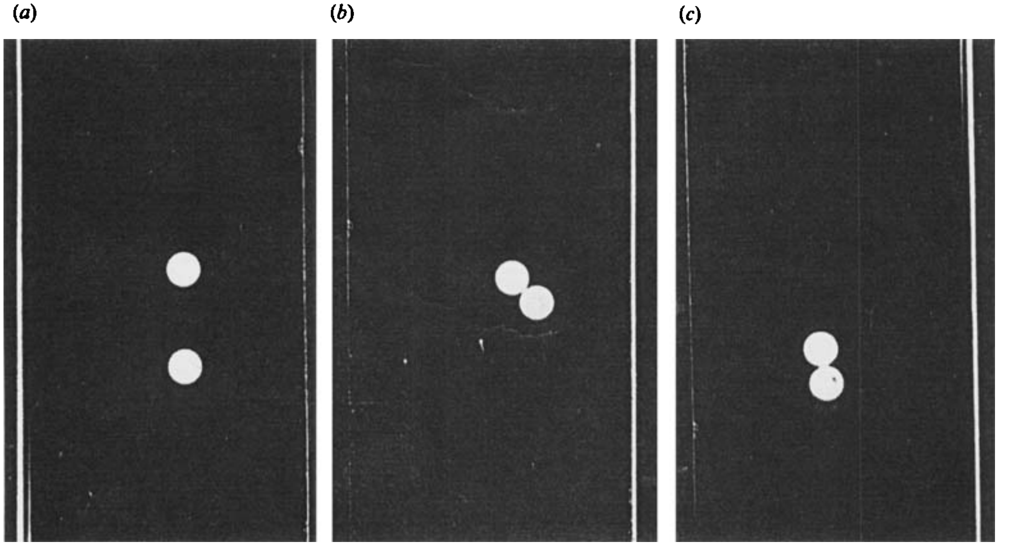


FIGURE 5. Drafting, kissing and tumbling in the vertical two-dimensional bed. The mechanisms are not affected by the inclination of the bed. $Re = 650$.

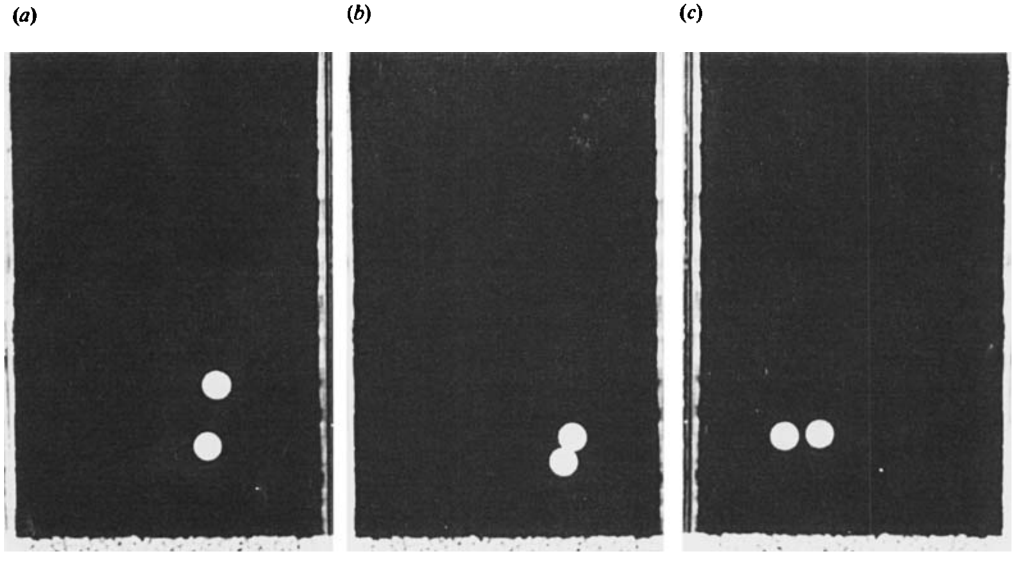


FIGURE 6. Drafting, kissing and tumbling in the three-dimensional bed. The mechanisms are not affected by the walls. $Re = 800$.

figures 4 (a, b) and 5 (a). The second sphere is very rapidly sucked into contact with the first, then kissed. This is shown in figures 4 (c) and 5 (b). The falling motion of contacting spheres aligned in the direction of motion is very unstable. As soon as they kiss, they tumble and are thrown apart as shown in figures 4 (d) and 5 (c). A slight angular displacement of the line of centres of two contacting spheres will induce an unsymmetric wake which evidently gives rise to a strong destabilizing couple. Drafting, kissing and tumbling can occur repeatedly and with great frequency.

(a) (b) (c)

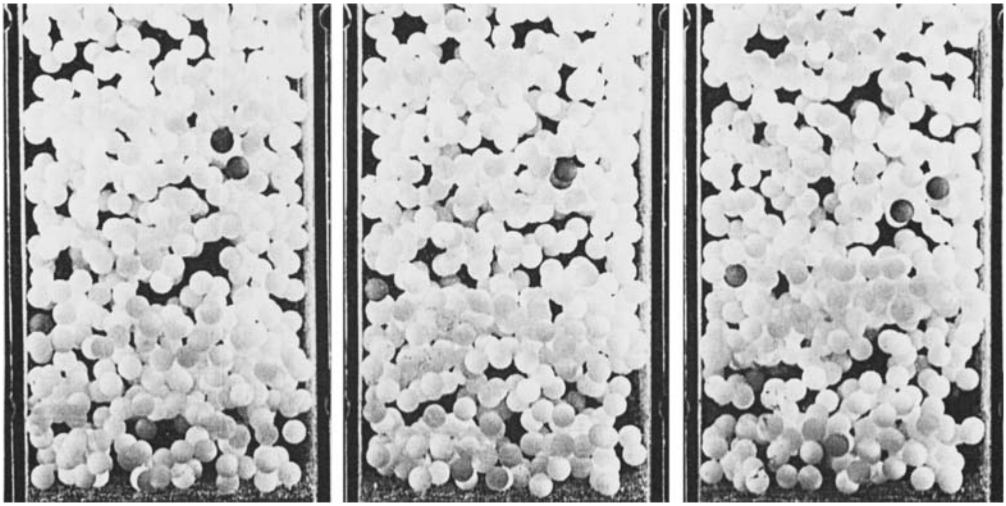


FIGURE 7. The interaction of two dyed plastic spheres in the three-dimensional apparatus. The stability of rising cross-stream arrays is also apparent. $Re = 850$.

Moreover, since the wake of spheres is determined by the direction of streaming, the events of this scenario are controlled locally and also occur in the three-dimensional bed, as can be seen in figure 6(*a–c*). The still-picture camera sequence of figure 7(*a–c*) taken at $\frac{1}{3}$ s intervals, shows the drafting, kissing and tumbling occurring with the dyed pair of plastic spheres on the right-hand side of the pictures, in the 3 in. × 3 in. cross-section channel.

The stabilizing effect of the wall friction accounts for the higher Re in the inclined channel, but the evidence shows that the mechanisms of drafting, kissing and tumbling are affected neither by inclination nor by wall effects. Generally speaking, the dynamics in the regions near the wall is not inertially dominated. Thus, the particle interactions near the wall could not be controlled by inertial suction, which contradicts blatantly the experimentally observed drafting of one sphere in the wake of another. Walls can have an important quantitative effect on frictional drag without altering the qualitative balance with opposing forces associated with inertial suction.

Since it appeared impossible to maintain a water flow of absolute uniformity over the cross-sectional area of the three-dimensional bed, the increased level of the fluid turbulence reduced the distance of interaction of the spheres. In fact, the shrinking of the wake region behind bluff bodies with increasing Reynolds number has been reported in the physical explanations of Gerrard (1966), as well as in the experimental work of Achenbach (1974). Turbulence does not overcome the capture phenomenon, though it does reduce the distance of interaction.

In figure 8(*a, b*) we have exhibited one more feature of fluidization seen in two-dimensional flows with many particles in purer form. The total number of spheres in figure 8(*a*) is eight; eight spheres can span the 2 in. width of the channel with some play. The picture shows seven spheres standing stationary in the upward stream of water. The eighth sphere is stationary, standing on the equalizing section. Figure 8(*b*) shows the same configuration in the vertical two-dimensional bed. They are stable, for a time, in the configuration in which their lines of centres are perpendicular to the stream. Since they rise, they are being accelerated in their wakes, pulled along

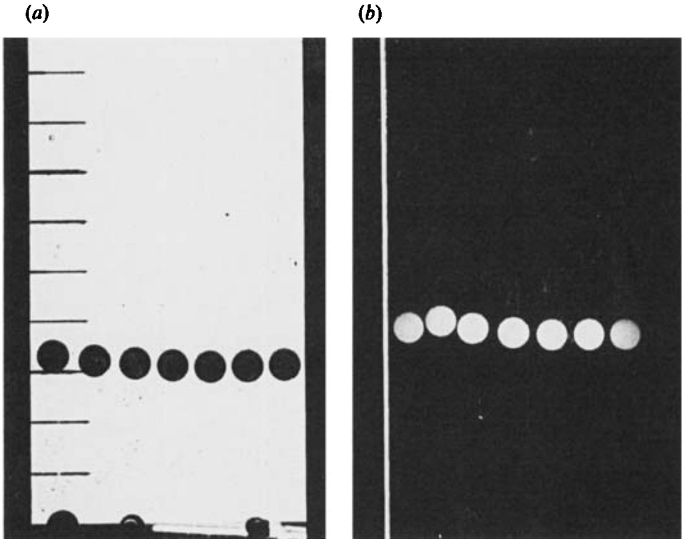


FIGURE 8. The spheres in the two-dimensional bed rise slowly in their own wakes. Different spacings are stable. There is no unique stable spacing. The arrangement is independent of the inclination of the bed. (a) Inclined channel. (b) Vertical channel.

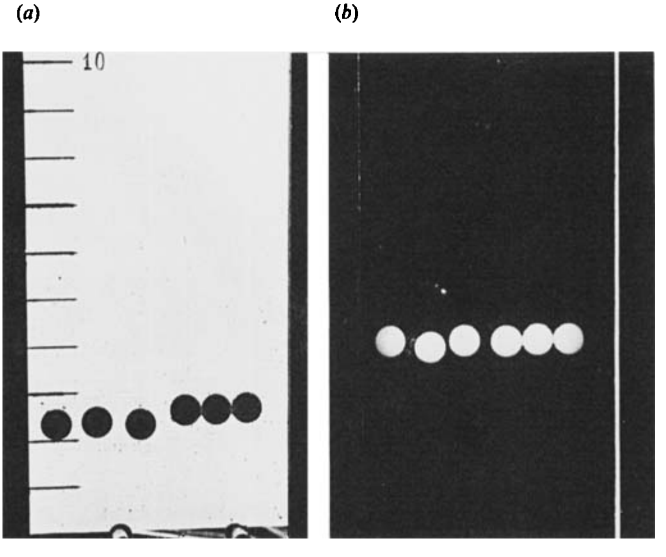


FIGURE 9. Three spheres cemented together line up with three other spheres cemented by their own wakes. The configuration is independent of the inclination of the bed. (a) Inclined channel. (b) Vertical channel.

by the suction. We call attention to the fact that spheres aligned cross-stream do not touch, but they are evidently spaced to produce stability. There are many different stable spacings. These cross-stream lines of spheres can be found in the many photographs of the two-dimensional beds of spheres shown in §5, especially figure 15(a, b).

To explore the stability of the cross-stream alignments, we decided to cement three spheres together. The cemented spheres are massively stable when their line of centres (*a*) (*b*) (*c*)

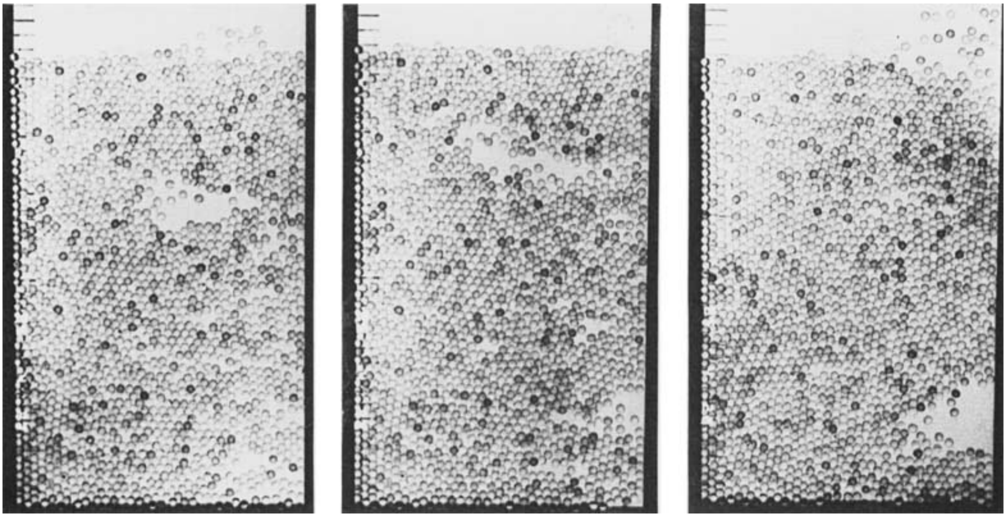


FIGURE 10. Fluidization of glass beads at 0.53 average voidage and $Re = 1080$ in the two-dimensional inclined bed. The channel is 8 in. wide. The motion of voids occurs as a withdrawal of spheres at the void roof and the collection of spheres at the void floor.

is perpendicular to the stream. In a striking display of this basic feature of fluidization, figure 9(*a*, *b*) shows that the stability of the cross-stream arrays is indeed unaffected by the inclination of the walls. The increased drag experienced by the spheres in this configuration (Tsuji *et al.* 1982) testifies to the fact that the cemented triplet and the free spheres rose abreast only when spaced in stable configurations. The factors which enter into the breakdown of these configurations are not perfectly understood. However, it was obvious that inter-particle collisions associated with drafting, kissing and tumbling and fluid turbulence play an important role. This breakdown may be more or less identified with the propagation of banded aggregates of rising cross-stream arrays of spheres which gain spheres from the tumbling of kissing spheres drafted into the aggregate from the top and lose spheres from the collapse of the aggregate from the bottom. This regulating mechanism for propagation of aggregates is evident in figures 7 to 14.

4. Experimental results for the fluidization of many spheres

Figure 10(*a–c*) shows the onset and propagation of void cracks in a channel 8 in. wide using glass beads in the inclined channel. They are separated by time intervals of about $\frac{1}{3}$ s. The cracking of closely packed structures is called void cracking and can be viewed as a density perturbation or even as a shock wave of density (Wallis 1962; Verloop *et al.* 1970; Fanucci *et al.* 1979). The average void fraction in these figures is 0.53 and $Re = 1080$. Under these conditions, the bed is nearly stationary and nearly packed. Essentially, the same structure can be seen in the vertical channel, although not with the same stability. We might expect that the pressure drop across the stationary closely packed bed is related to the superficial velocity by the well-known formula for the pressure drop–velocity relation in the state of incipient fluidization, like the Ergun equation (Richardson 1971).

The effects of inertia of the liquid in flow through a porous media appear as a drag

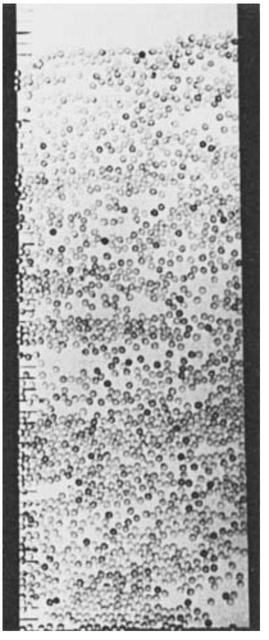


FIGURE 11. Fluidization of glass beads of 0.72 average voidage and $Re = 1800$ in the two-dimensional bed. The channel is 8 in. wide. Propagation of void cracks and of stable cross-stream arrays are both evident in the figure.

proportional to the square of the velocity. The idea is that this drag is due to the 'dead water' region behind particles (Joseph, Nield & Papanicolau (1982) derive the quadratic drag (1) from this idea). The same idea seems to operate in fluidized beds, i.e. the contributions of the inertial hydrodynamic interaction between particles to the bulk properties of the flow can be expressed as

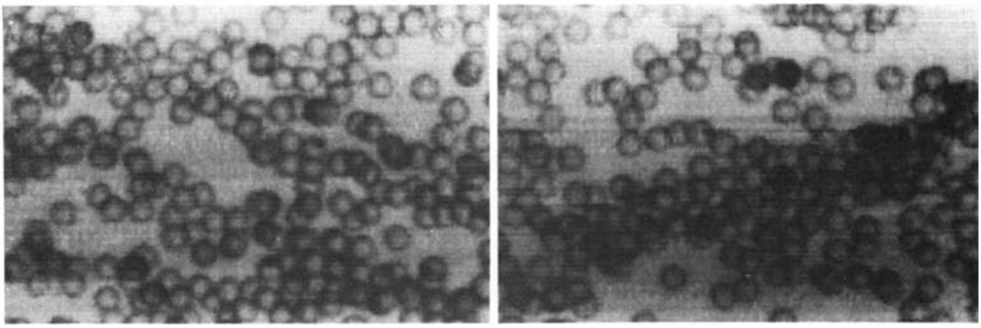
$$f = g(\phi)\,|\boldsymbol{u}|\boldsymbol{u}, \tag{1}$$

where u is the fluid velocity relative to the particle and $g(\phi)$ is an unknown function of the voidage ϕ. Although a discussion of constitutive relations is beyond the scope of the present work, the quadratic drag term suggested here is physically consistent with the observed phenomena.

Packed beds of fluidized spheres are different from porous material because the drag forces may move the spheres. This type of local density perturbation usually takes form as void cracking of the packed structure. The upper surface of the crack moves upward, widening the crack. This type of cracking occurred only in the two-dimensional beds. The cracks are always perpendicular to the bulk flow direction, more or less, and never parallel to the flow. The flow across the roof of the crack thus creates a local pressure gradient that drives the packed structure above the crack upwards. The relief of this pressure gradient in the final bursting of the ascending void crack in the upper layer of the bed shown in figure 10(c) is indicative of its presence as a driving force. The existence of this local pressure gradient is consistent with the more general observation of Harrison & Davidson (1963, 1971) that for a fluidized bed of given voidage the pressure drop at a given velocity is less than for a randomly packed fixed bed at the same velocity and voidage.

Another mechanism for the propagation of cracks is the continuous dislodging of

(a) (b)



(c)

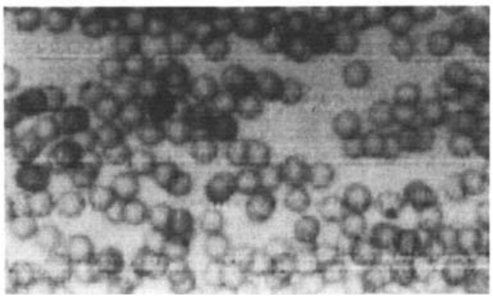


FIGURE 12. Close-up of the high-speed video camera sequence of fluidization of 0.117 in. diameter glass beads in the vertical two-dimensional 6 in. channel. The average voidage is 0.88. The horizontal travelling-wave structure from bottom to top is clearly associated with the crosswise arrangements. The streamwise doublets can break down the waves. $Re = 500$.

spheres from the void roof. This occurred whenever there was a local defect in the otherwise stable lattice of the aggregate. The rising velocities of the void roof which propagates by depletion of spheres from the roof were always smaller than the velocity of single particles in free fall and the velocity of the flow across the crack was never sufficient to keep the dislodged particles fluidized.

A further expansion of the bed is shown in figure 11 with $Re = 1800$, $\phi = 0.72$. This expansion is achieved by a wavelike motion in which aggregates clustered around lines perpendicular to the direction of flow rise in their own wakes. These wavelike motions are revealed in the sequence of figure 12(*a–c*) taken with the high-speed video camera at a rate of 1000 frames per second. The beads were glass spheres 0.117 in. diameter. From bottom to top the uprising motion of the arrays combined with the breaking down of the falling streamwise pairs is again the dominating feature. The pictures are evenly spaced in time intervals of 0.1 s. We claim, as an observation of general validity, that the interplay of upwards drifting of crosswise alignments in their own wakes with falling of streamwise pairs with rearward spheres catching forward spheres as they are accelerated by inertial suction of the wake is the governing mechanism underway.

Figure 13(*a*, *b*) shows fluidization of glass beads in the 3 in. wide inclined channel with $Re = 1250$. The presence of large voids, the big clear-water regions, is not well described by specifying a void fraction. The slugging flow of packed spheres displaces regions of clear water. The same cracks or shocks observed in the wider channel now

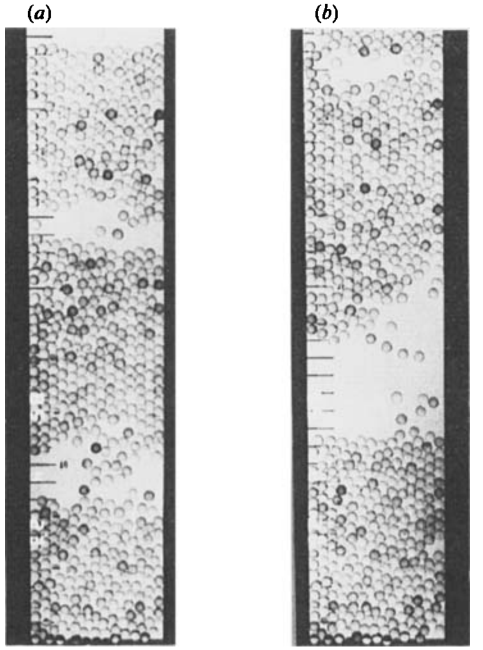


FIGURE 13. Fluidization of glass beads at $Re = 1250$. The same cracking mechanism and expansion can be observed here in a larger scale. The beads are 0.231 in. diameter in the two-dimensional inclined 3 in. channel.

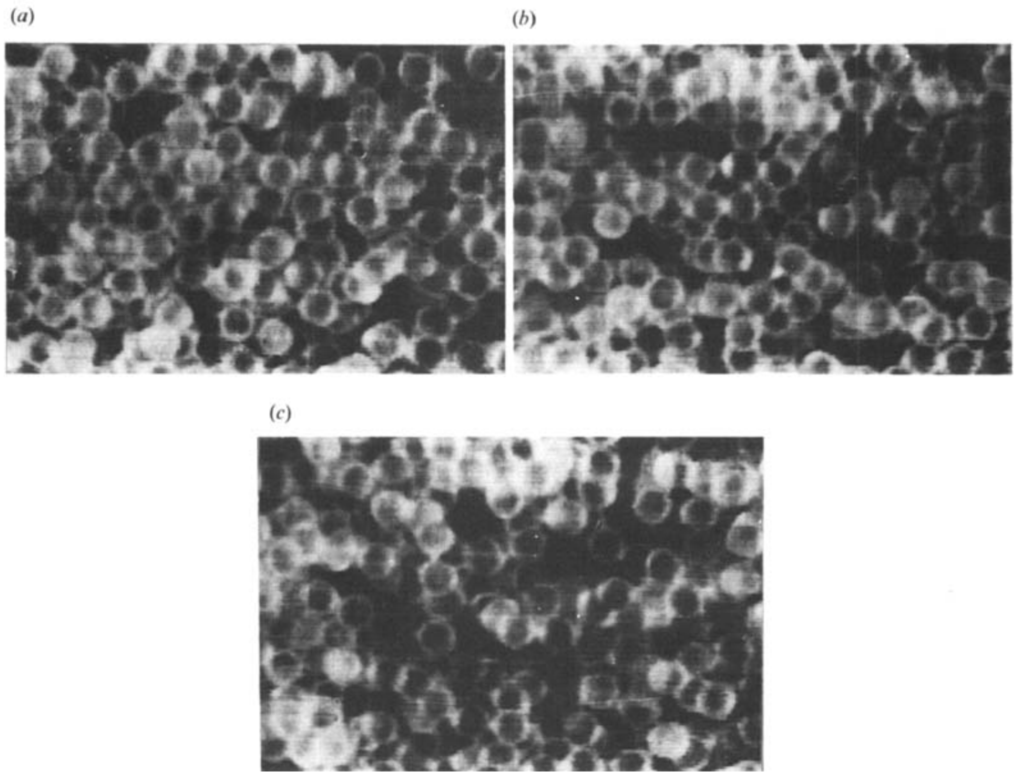


FIGURE 14. Close-up of the high-speed video camera sequence of fluidization of the 0.117 in. diameter glass spheres in the three-dimensional bed. The average voidage is 0.90. $Re = 690$.

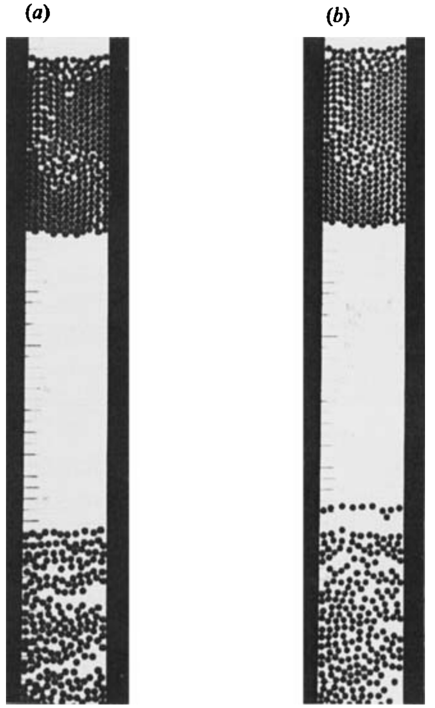


FIGURE 15. Fluidization of plastic beads at $Re = 290$ in the inclined two-dimensional 3 in. wide bed. The spheres at the top of the bed are stable and close packed with contacting vertical lines of centres, as in figure 1. The bottom configuration with stable single cross-stream row is nearly stationary.

span the whole channel. Also, the expansion mechanism, as described before, is revealed here in the same way. The different sequences taken with the high-speed video camera shown in figure 14 (*a*–*c*), separated by time intervals of 0.05 s, reveal the distinctive features referred to thus far. First, the characteristic topological structure of closely packed spheres whose line of contact lies along the stream, as is vividly depicted in the aggregates at the top of figure 15 (*a*, *b*), manifests itself in the preferential way the spheres collide with one another, i.e. the top sphere collides with the bottom one following in its wake. This topological feature is compatible with the dynamics associated with drafting and kissing. The lateral collisions can be attributed to the eddying motion of the liquid, since there is no possible lateral attraction between two spheres, as pointed out earlier. Secondly, even for these highly turbulent flows, the uprising motion is entirely determined by cross-stream arrays of spheres. The fluid is pushed upward through the channels around the vertical line of centres of contacting spheres. There is a wake above each sphere, providing the glue. Isolated particles drift into these stream channels and are dragged into their own wakes, gluing more spheres to form the aggregates.

The remarkable line of particles at the top of the bottom part of the beds shown in figure 15 (*a*, *b*) was a characteristic feature of the flow in both the vertical and inclined channels.

Further increase of the Reynolds number does not change the basic mechanisms, and although the eddying motion sets up the lateral collisions of the particles, the expanded bed reveals essentially the same structure.

5. Conclusions

The following results are suggested by the experiments discussed in this paper.

1. The dynamics of beds of particles fluidized by water at moderate and high Reynolds numbers is dominated by local mechanisms associated with wakes.

2. The results of Happel & Pfeffer (1960) indicate that even at Reynolds numbers as small as $\frac{1}{2}$ the fluidization of spheres is strongly influenced by nonlinear effects of wakes and cannot be described by models based in Stokes equations.

3. Drafting, kissing and tumbling appears to be the major rearrangement mechanism in the fluidization of beds of spheres with water.

4. Long particles and long arrays of single particles line up broadside onto the stream. The instability of streamwise pairs of kissing particles and the relative stability of crosswise pairs (doublets) seem to play a major role in aggregation.

5. Drafting particles fall and cross-stream arrays of particles rise in a stream in which single particles are neutrally buoyant.

6. Cross-stream arrays in beds of spheres constrained to move in two dimensions show great stability. Single horizontal lines of rising particles, even with different spacings, are robustly stable in the vertical and inclined channel. Single stable stationary lines of particles in horizontal arrays and stable aggregates of close-packed spheres separated by regions of clear water were observed only in the inclined channel.

7. A weaker form of cooperative motion of cross-stream arrays of rising spheres is found in beds of square cross-section where the spheres may move freely in three dimensions. Horizontal arrays, which have the greatest drag, seem to be a fundamental feature of fluidization at moderate and high Reynolds numbers.

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REFERENCES

ACHENBACH, E. 1974 Vortex shedding from spheres. *J. Fluid Mech.* **62**, 209–221.

EL-KAISSY, M. M. & HOMSY, G. M. 1976 Instability waves and the origin of bubbles in fluidized beds. *Intl J. Multiphase Flow* **2**, 379–395.

FANUCCI, J. B., NESS, N. & YEN, R. H. 1979 On the formation of bubbles in gas-particulate fluidized beds. *J. Fluid Mech.* **94**, 353–367.

GARSIDE, J. & AL-DIBOUNI, M. 1973 Behavior of liquid fluidized beds containing a wide size distribution of solids. *Fluidization and its Applications*, pp. 53–62. Toulouse.

GERRARD, J. H. 1966 The mechanics of the formation region of vortices behind bluff bodies. *J. Fluid Mech.* **25**, 401–413.

HAPPEL, J. & PFEFFER, R. 1960 The motion of two spheres following each other in a viscous fluid. *AIChE J.* **6**, 129–133.

HARRISON, D. & DAVIDSON, J. 1963 *Fluidized Particles*, pp. 15–19. Cambridge University Press.

HARRISON, D. & DAVIDSON, J. 1971 *Fluidization*, pp. 47–50. Academic.

HOMSY, G. M., EL-KAISSY, M. M. & DIDWANIA, A. 1980 Instability waves and the origin of bubbles in fluidized bed. Part 2. *Intl J. Multiphase Flow* **6**, 305–318.

JOSEPH, D. D., NIELD, D. A. & PAPANICOLAU, G. 1982 Nonlinear equation governing flow in a saturated porous medium. *Water Res. J.* **18**,. 1049–1052; Erratum 1983 **19**, 591.

KAYE, B. H. & BOARDMAN, R. P. 1962 Cluster formation in dilute suspensions. *Symp. Interact. Fluids and Particles*, p. 17. London.

LAMB, H. 1938 *Hydrodynamics*, p. 191. Dover.

LIU, J. T. C. 1982 Notes on a wave-hierarchy interpretation of fluidized bed instabilities. *Proc. R. Soc. Lond.* A **380**, 229–239.

LIU, J. T. C. 1983 *Nonlinear instabilities in fluidized beds. Mechanics of Granular Materials: New Models and Constitutive Relations* (ed. J. T. Jenkins & M. Satake), pp. 357–364.

OSEEN, C. W. 1927 *Hydrodynamik*, pp. 199 ff. Akademische Verlagsgesellschaft M.B.H.

RICHARDSON, J. F. 1971 Incipient Fluidization and Particulate Systems. In *Fluidization* (ed. J. F. Davidson & D. Harrison), pp. 50–51. Academic.

RICHARDSON, J. F. & MEIKLE, R. A. 1964 Sedimentation and fluidization. Part 4. Drag force on individual particles in an assemblage. *Trans. Instn Chem. Engrs* **39**, 357–362.

RICHARDSON, J. F. & ZAKI, W. 1954 Sedimentation and fluidization. Part 1. *Trans. Instn Chem. Engrs* **32**, 35–53.

ROWE, P. N. 1961 Drag forces in a hydraulic model of a fluidized bed. Part 2. *Trans. Instn Chem. Engrs* **39**, 175.

ROWE, P. N. & HENWOOD, G. A. 1961 Drag forces in a hydraulic model of a fluidized bed. Part 1. *Trans. Instn Chem. Engrs* **39**, 43.

TSUJI, Y., MORIKAWA, Y. & TERASHIMA, K. 1982 Fluid-dynamic interaction between two spheres. *Intl J. Multiphase Flow* **8**, 71–78.

VERLOOP, J. & HEERTJES, P. M. 1970 Shock waves as a criterion of the transition from homogeneous to heterogeneous fluidization. *Chem. Engng Sci.* **25**, 825.

VOLPICELLI, G., MASSIMILLA, L. & ZENZ, F. S. 1966 Nonhomogeneities in solid–liquid fluidization. *Chem. Engng Symp. Ser.* no. 62, 67 and 42.

WALLIS, G. B. 1962 A simplified one-dimensional representation of two-component vertical flow and its applications to batch sedimentation. *Symp. Interact. Fluids and Particles, London*, vol. 9.

WILHELM, R. H. & KWAUK, M. 1948 Fluidization of solid particles. *Chem. Engng Prog.* **44**, 201.